Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name: Metric Space

Subject Code: 4SC05MSC1 **Branch: B. Sc. (Mathematics)**

Semester: 5 **Date: 07/12/2015** **Time:02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Define: Metric Space.

b) Define: Derived Set.

c) Define Cauchy sequence in a metric space.

d) If $A = [2, 7]\ U\ [7,9]$, then find$\bar{A}$.

e) Define: Complete metric space.

f) Is $[5, 6]$compact?

g) Define dense sets in a metric space.

h) If $= R, A = Q$, then find $int\ A$.

i) Define: Diameter of a non-empty set.

j) $\phi$and$X$ are open in any metric. Determine whether the statement is True or False?

k) The arbitrary union of closed sets in a metric space is closed set. Determine whether the statement is True or False?

l) Every Cauchy sequence is convergent. Determine whether the statement is True or False?

m) Continuous image of compact set is compact. Determine whether the statement is True or False?

n) The cantor set is perfect set. Determine whether the statement is True or False?

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Let (X, d) be any metric space. Show that the function d_1 defined by $d_1(x, y) = \frac{d(x,y)}{1+d(x,y)}$, $\forall\, x, y \in X$ is a metric on X . **(05)**

b) Which of the following sets are open sets. Justify. **(05)**
$i)$ $(-1,1)$ on R,
$ii)$ $\{ (x, y) / x = y \}$ on R^2,
$iii)$ $\{(x, y) / x^2 + y^2 < 1 \}$ on R^2,
$iv)$ $[0, 1)$ on R,
$v)$ $(0, 1)\ U\ (2, 3)$.

c) Let (X, d) be any metric space. Prove that a subset F of X is closed if and only if its complement in X is open. **(04)**

Q-3 **Attempt all questions** **(14)**

a) Prove that every compact subset A of a metric space (X, d) is bounded. **(05)**

b) Let (X, d) be a metric space and $Y \subseteq X$, then prove that a subset to be open in (Y, d_Y) if and only if there exists a set G open in (X, d) such that $A = G \cap Y$. **(05)**

c) Let (X, d) be a metric space, and let $d^{'}(x, y) = \min\{1, d(x, y)\}$ for all $x, y \in X$. Then show that d and d' are equivalent. **(04)**

Q-4 **Attempt all questions** **(14)**

a) Prove that the function $d: C \times C \to R$ defined by $d(x, y) = \frac{2|x-y|}{\sqrt{1+|x^2|}\sqrt{1+|y^2|}}$ is a metric on the set of all complex numbers. **(05)**

b) Which of the following sets are closed sets. Justify. **(05)**
$i)$ $[-1,1]$ on R,
$ii)$ $\{ (x, y) / x = y \}$ on R^2,
$iii)$ $\{(x, y) / x^2 + y^2 = 1 \}$ on R^2,
$iv)$ $\{(x, y) / x^2 + y^2 > 1 \}$ on R^2,
$v)$ $\{ 1,2,3,4,5\}$.

c) Prove that every open sphere is open set. **(04)**

Q-5 **Attempt all questions** **(14)**

a) Let A and B be any two subsets of a metric space (X, d). Then prove that **(05)**

$i) \overline{A \cup B} = \bar{A} \cup \bar{B}$, $ii) \overline{A \cap B} \subseteq \bar{A} \cap \bar{B}$.

	b)	Prove that arbitrary union of open sets is open.	**(05)**
	c)	Let A and B be any two subsets of a metric space (X, d), then prove that A is closed if and only if $A \supseteq Fr(A)$.	**(04)**
Q-6		**Attempt all questions**	**(14)**
	a)	State and prove Hein-Borel theorem.	**(07)**
	b)	Let (X, d_1) and (Y, d_2) is metric spaces. Prove that $f: X \to Y$ is continuous if and only if $f(\bar{A}) \subseteq \overline{f(A)}$, for every $A \subseteq X$.	**(05)**
	c)	Let A and B be any two subsets of a metric space (X, d), then prove that $A \subseteq B$implies$int\ A \subseteq int\ B$.	**(02)**
Q-7		**Attempt all questions**	**(14)**
	a)	For any non-empty subset A of a metric space (X, d) the function $f: X \to R$ given by $f(x) = d(x, A)$, for $x \in X$ is uniform continuous. Also prove that $f(x) = 0$ if and only if$x \in \bar{A}$.	**(07)**
	b)	Prove that continuous image of a connected set is connected.	**(05)**
	c)	If $f(x) = x^2$, $0 \le x \le \frac{1}{3}$, then show that f is contracting mapping on $\left[0, \frac{1}{3}\right]$ with the usual metric d.	**(02)**
Q-8		**Attempt all questions**	**(14)**
	a)	State and prove Banach Fixed point theorem.	**(07)**
	b)	Let (X, d_1) and (Y, d_2) is metric space and f is a function from Xinto Y. Then prove that f is continuous at $a \in X$if and only if for every sequence $\{a_n\}$ converging to a, the sequence $\{f(a_n)\}$ should be converges to $f(a)$.	**(05)**
	c)	Find $int\ A$, $ext\ A$, $fr\ A$,$bd\ A$ for the set $X = N$, $A = \{1, 2, 3, 4, 5, 6\}$.	**(02)**